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A NON-LOCAL PROBLEM WITH DISCONTINUOS GLUING CONDITION FOR A LOADED WAVE-DIFFUSION EQUATION INVOLVES FRACTIONAL DERIVATIVE.

ABSTRACT: In this work an existence and uniqueness of solution of non-local boundary value problem with discontinuous matching condition for the loaded parabolic-hyperbolic equation involving the Riemann-Liouville fractional derivative have been investigated. The uniqueness of solution is proved by the method of integral energy and the existence is proved by the method of integral equations.

Annotatsiya: Ushbu ishda Rimann-Liuvill kasr hosilasi ishtirokidagi yuklangan parabolik-giperbolik tenglama uchun uzlukli ulash shartli nolokal chegaraviy masala yechimining mavjudligi va yagonaligi oʻrganildi. Yechimning yagonaligi integral energiya usuli bilan, mavjudligi esa integral tenglamalar usuli bilan isbotlangan.

Key words and phrases: Loaded equation, wave-diffusion equation, Riemann-Liouville fractional derivative, existence and uniqueness of solution, non-local condition, discontinuous matching condition, integral energy, integral equations..

Introduction.

Notice, that the modeling of many phenomena in various fields of science and engineering reduce to the fractional differential. We can find numerous applications in viscoelasticity, neurons, electrochemistry, control, porous media, electromagnetism, etc., (see [1–6]). There has been significant development in fractional differential equations in recent years; see the monographs of A.A. Kilbas, H.M. Srivastava, J.J. Trujillo [7], K.S.Miller and B.Ross [8], I.Podlubny [9], S.G. Samko, A.A. Kilbas , O.I. Marichev. [10] and the references therein.

Very recently some basic theory for the initial boundary value problems of fractional differential equations involving a Riemann-Liouville differential operator of order $0<\alpha\leq 1$ has been discussed by L. Lakshmikantham and A.S. Vatsala [11, 12]. In a series of papers (see [13, 14]) the authors considered some classes of initial value problems for functional differential equations involving Riemann-Liouville and Caputo fractional derivatives of order $0<\alpha\leq 1$: For more details concerning geometric and physical interpretation of fractional derivatives of Riemann-Liouville and Caputo types [15].

There are many works [16-19], devoted to the studying of boundary value problem (BVP)s for parabolic-hyperbolic equations, involving fractional derivatives. BVPs for the mixed type equations involving the Caputo and the Riemann-Liouville fractional differential operators were investigated in works [20-22].

This paper deals the existence and uniqueness of solution of the non-local problem with discontinuous matching condition for loaded mixed type equation involving the Riemann-Liouville fractional derivative.

Problem formulation.

We consider the equation:

$$0=\begin{cases} u_{xx}-D_{oy}^{\alpha}u+p(x,y,u), & \text{at } y>0 \\ u_{xx}-u_{yy}+q(x,y,u), & \text{at } y<0 \end{cases} \quad (1)$$

with operation [1],[22]:

$$D_{oy}^{\alpha}u=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dy}\int_0^y (y-t)^{-\alpha}u(x,t)dt \,, \tag{2}$$

$$p(x,y,u)=p_1(x,y)\lim_{y\to 0}\int_0^y (y-t)^{-\alpha}u(x,t)dt \,, \tag{3}$$

$$q(x,y,u)=q_1(x,y)\int_0^{x-y}(x-y-t)^{\delta-1}u(t,-0)dt \,. \tag{4}$$

Let us, Ω is domain, bounded with segments: $A_1A_2=\{(x,y):\ x=1,\ 0<y<h\}$, $B_1B_2=\{(x,y):\ x=0,\ 0<y<h\}$, $B_2A_2=\{(x,y):\ y=h,\ 0<x<1\}$ *at the* $y>0$, and characteristics : $A_1C: x-y=1$; $B_1C: x+y=0$ of the equation (1) at $y<0$, where $A_1(1;0), A_2(1;h), B_1(0;0), B_2(0;h), C\left(\frac{1}{2};-\frac{1}{2}\right)$.

We enter designations: $\theta(x)=\theta\left(\frac{x}{2},-\frac{x}{2}\right)$.

$\Omega^+=\Omega\cap(y>0)$, $\Omega^-=\Omega\cap(y<0)$, $I_1=\left\{x:\frac{1}{2}<x<1\right\}$, $I_2=\{y:0<y<h\}$.

In the domain of Ω the following problem is investigated.

Problem I. To find a solution $u(x,y)$ of the equation (1) from the following class of functions:

$W=\left\{u:\ D_{0y}^{\alpha-1}u(x,y)\in C(\bar{\Omega}^+), u(x,y)\in C(\bar{\Omega}^-)\cap C^2(\Omega^-), u_{xx}\in C\left(\Omega^+\right), D_{oy}^{\alpha}u\in C\left(\Omega^+\cup I\right)\right\}$

satisfies boundary conditions:

$$u(x,y)\Big|_{A_1A_2}=\varphi_1(y),\ 0\le y\le h,\quad u(x,y)\Big|_{B_1B_2}=\varphi_2(y),\ 0\le y\le h, \tag{5}$$

$$\frac{d}{dx}u\left(\theta(x)\right)=a(x)u_y(x,-0)+b(x)u_x(x,-0)+c(x)u(x,-0)+d(x),\ \ x\in I_1. \tag{6}$$

and gluing conditions:

$$\lim_{y\to+0} y^{1-\alpha}u(x,y)=\mu(x)u_y(x,-0), \tag{7}$$

$$\lim_{y\to+0} y^{1-\alpha}\left(y^{1-\alpha}u(x,y)\right)_y=\lambda(x)u_y(x,-0),\ (x,0)\in A_1B_1 \tag{8}$$

where $\varphi_i(y)$, $a(x)$, $b(x), c(x), d(x)$, $\mu(x)$ and $\lambda(x)$ are given functions, besides $\mu(x),\lambda(x)\neq 0\ \ \forall x\in[0,1]$.

Method of investigation.

In order to complete prove well-posedness (correctness) of the formulated, required to uniqueness and existence of solution of the posed problem. Uniqueness of the Problem I is proved using by the method of integral energy and the existence is proved by the method of integral equations.

We assume, that $q_1(x,y) = -\tilde{q}(x+y)\cdot q^*(x-y)$, then the equation (1) at $y \le 0$ on the characteristics coordinate $\xi = x+y$ and $\eta = x-y$ totally looks like:

$$u_{\xi\eta} = \frac{1}{4}\tilde{q}\ (\xi)q^*(\eta)\int_0^{\eta}\frac{u(t,-0)}{(\eta-t)^{1-\delta}}dt.$$

Let's enter designations: $u(x,-0) = \tau^-(x),\ 0 \le x \le 1;\ u_y(x,-0) = \nu^-(x), 0 < x < 1$.

Known, that solution of the Cauchy problem for the equation (1) in the domain of Ω^- can be represented as follows:

$$u(x,y) = \frac{\tau\ (x+y)+\tau\ (x-y)}{2} - \frac{1}{2}\int_{x+y}^{x-y}\nu^-(t)dt +$$

$$+\frac{1}{4}\int_{x+y}^{x-y}q^*(\eta)d\eta\int_{x+y}^{\eta}\tilde{q}(\xi)d\xi\int_0^{\eta}\frac{\tau^-(t)}{(\eta-t)^{1-\delta}}dt. \quad (9)$$

After using condition (6) and taking (3) into account from (9) we will get:

$$\left(2a(x)+1\right)\nu^-(x) = \frac{1}{2}q^*(x)\int_0^x\tilde{q}(\xi)d\xi\int_0^x\frac{\tau^-(t)}{(x-t)^{1-\delta}}dt +$$

$$+\left(1-2b(x)\right)\tau'^-(x) - 2c(x)\tau^-(x) - 2d(x) \quad (10)$$

Functional relation (10) we can rewrite as:

$$\left(2a(x)+1\right)\nu^-(x) = Q(x)\int_0^x\frac{\tau^-(t)}{(x-t)^{1-\delta}}dt + \left(1-2b(x)\right)\tau'^-(x) - 2c(x)\tau(x) - 2d(x),$$

where $Q(x) = \dfrac{q^*(x)}{2}\displaystyle\int_0^x\tilde{q}(\xi)d\xi$.

Considering designations $\tau^+(x) = \lim\limits_{y\to 0}D_{0y}^{\alpha-1}u(x,y)$, $\lim\limits_{y\to+0}y^{1-\alpha}\left(y^{1-\alpha}u(x,y)\right)_y = \nu^+(x)$ and $\lim\limits_{y\to 0}D_{0y}^{\alpha-1}f(y) = \Gamma(\alpha)\lim\limits_{y\to 0}y^{1-\alpha}f(y)$ gluing condition (7), (8) we can rewrite as

$$\tau^+(x) = \Gamma(\alpha)\mu(x)\tau^-(x),\qquad \nu^+(x) = \lambda_1(x)\nu^-(x). \quad (11)$$

Further from the Eq. (1) at $y \to +0$ owing to account (2), (11) we get [20]:

$$\mu(x)\tau''(x) - \Gamma(\alpha)\lambda(x)\nu^-(x) + (\mu(x)\tilde{p}(x) + \mu''(x))\tau(x) = 0, \quad (12)$$

where $\tau(x) \equiv \tau^-(x)$, $\tilde{p}(x) = \lim\limits_{y\to 0}D_{0y}^{\alpha-1}p_1(x,y)$.

Main Result-1. The Uniqueness of solution of the Problem *I*.

Theorem 1. If satisfies conditions

$$\frac{Q(0)}{2a(0)+1}\frac{\lambda(0)}{\mu(0)} \ge 0,\ \ \tilde{p}(x) + \frac{\mu''(x)}{\mu(x)} \ge 0, \quad (13)$$

$$\left(\frac{Q(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\right)' \ge 0,\ \frac{c(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)} \le 0,\ \left(\frac{1-2b(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\right)' \le 0 \quad (14)$$

then, the solution $u(x,y)$ of the Problem I is unique.

Proof. Known, that, if homogeneous problem has only trivial solution, then we can state that original problem has unique solution. For this aim we assume that the Problem I has two solutions, then denoting difference of these as $u(x,y)$ we will get appropriate homogenous problem.

Equation (12) we multiply to $\tau(x)$ and integrated from 0 to 1:

$$\int_0^1 \tau''(x)\tau(x)dx - \Gamma(\alpha)\int_0^1 \frac{\lambda(x)}{\mu(x)}\tau(x)\nu^-(x)dx + \int_0^1 \tau^2(x)\tilde{p}_1(x)dx = 0, \qquad (15)$$

Where $\tilde{p}_1(x) = \dfrac{\mu(x)\tilde{p}(x) + \mu''(x)}{\mu(x)}$

We will investigate the integral

$$I \equiv \Gamma(\alpha)\int_0^1 \frac{\lambda(x)}{\mu(x)}\tau(x)\nu^-(x)dx - \int_0^1 \tau^2(x)\tilde{p}_1(x)dx = 0.$$

Taking (10) into account $d(x)=0$ we get:

$$I = \frac{\Gamma(\alpha)\Gamma(\delta)}{2}\int_0^1 \frac{Q(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\tau(x)D_{0\,x}^{-\delta}\tau(x)dx + \Gamma(\alpha)\int_0^1 \frac{1-2b(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\tau(x)\tau'(x)dx -$$

$$-2\Gamma(\alpha)\int_0^1 \frac{c(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\tau^2(x)dx - \int_0^1 \tau^2(x)\tilde{p}_1(x)dx =$$

$$= \frac{\Gamma(\alpha)}{2}\int_0^1 \frac{Q(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\tau(x)dx\int_0^x (x-t)^{\delta-1}\tau(t)dt + \frac{\Gamma(\alpha)}{2}\int_0^1 \frac{1-2b(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}d\left(\tau^2(x)\right) -$$

$$-2\Gamma(\alpha)\int_0^1 \frac{c(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\tau^2(x)dx - \int_0^1 \tau^2(x)\tilde{p}_1(x)dx. \qquad (16)$$

Considering $\tau(1)=0, \tau(0)=0$ (which deduced from the conditions (4), (5) in homogeneous case) and on a base of the formula [23]:

$$|x-t|^{-\gamma} = \frac{1}{\Gamma(\gamma)cos\frac{\pi\gamma}{2}}\int_0^\infty z^{\gamma-1}cos\left[z(x-t)\right]dz, \quad 0<\gamma<1$$

after some simplifications from (16) we will get:

$$I = \frac{\Gamma(\alpha)Q(0)}{4\Gamma(1-\delta)\sin\frac{\pi\delta}{2}(2a(0)+1)}\frac{\lambda(0)}{\mu(0)}\int_0^\infty z^{-\delta}\left[\left(\int_0^1 \tau(t)\cos ztdt\right)^2 + \left(\int_0^1 \tau(t)\sin ztdt\right)^2\right]dz +$$

$$+\frac{\Gamma(\alpha)}{4\Gamma(1-\delta)\sin\frac{\pi\delta}{2}}\int_0^\infty z^{-\delta}dz\int_0^1 \frac{\partial}{\partial x}\left[\frac{\lambda(x)}{\mu(x)}\frac{Q(x)}{2a(x)+1}\right]\left[\left(\int_0^x \tau(t)\cos ztdt\right)^2 + \left(\int_0^x \tau(t)sinztdt\right)^2\right]dx -$$

$$-\frac{\Gamma(\alpha)}{2}\int_0^1 \tau^2(x)\left(\frac{\lambda(x)}{\mu(x)}\frac{1-2b(x)}{2a(x)+1}\right)' dx - 2\Gamma(\alpha)\int_0^1 \frac{c(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)}\tau^2(x)dx -$$

$$-\int_0^1 \tau^2(x)\tilde{p}_1(x)dx. \qquad (17)$$

Thus, owing to (13),(14) from (17) it is concluded, that $\tau(x)\equiv 0$. Hence, based on the solution of the first boundary problem for the Eq.(1) [21],[26] owing to account (4) and (5) we will get $u(x,y)\equiv 0$ in $\overline{\Omega}^+$.Further, from functional relations (10), taking into account $\tau(x)\equiv 0$ we get that $\nu^-(x)\equiv 0$. Consequently, based on the solution (9) we obtain $u(x,y)\equiv 0$ in closed domain $\overline{\Omega}^-$.

Main Result-2. The existence of solution of the Problem I.

Theorem 2. If satisfies conditions (13), (14) and

$$y^{1-\alpha}\varphi_i(y)\in C\left(\overline{I_2}\right)\cap C^1\left(I_2\right)\ (i=1,2),\ q_1(x,y)\in C\left(\overline{\Omega^-}\right),\ p_1(x,0),Q(x)\in C\left(\overline{A_1B_1}\right)\cap C^2\left(A_1B_1\right), \qquad (18)$$

$$\lambda(x)\in C\left(\overline{I_1}\right)\cap C^1\left(I_1\right),\ \mu(x)\in C\left(\overline{I_1}\right)\cap C^2\left(I_1\right),\ a(x),\ b(x),\ c(x),\ d(x)\in C^1\left(\overline{I_1}\right)\cap C^2\left(I_1\right) \qquad (19)$$

then the solution of the investigating problem exists.

Proof. Taking (10) into account from Eq. (12) we will obtain

$$\tau''(x)-A(x)\tau'(x)=f(x)-B(x)\tau(x) \qquad (20)$$

where

$$f(x)=\frac{\Gamma(\alpha)\Gamma(\delta)\lambda(x)Q(x)}{2(2a(x)+1)\mu(x)}D_{0x}^{-\delta}\tau(x)+\frac{2\Gamma(\alpha)d(x)}{2a(x)+1}\frac{\lambda(x)}{\mu(x)} \qquad (21)$$

$$A(x)=\frac{\Gamma(\alpha)(1-2b(x))}{2a(x)+1}\frac{\lambda(x)}{\mu(x)},\quad B(x)=\frac{2\Gamma(\alpha)c(x)}{1+2a(x)}\frac{\lambda(x)}{\mu(x)}+\tilde{p}_1(x) \qquad (22)$$

Solution of the equation (20) together with conditions

$$\tau(0)=\varphi_1(0),\quad \tau(1)=\varphi_2(0) \qquad (23)$$

has a form

$$\tau(x)=A_1(x)\left(\int_0^x\left(B(t)\tau(t)-f(t)\right)A_1'(t)dt+\frac{\varphi_1(0)-\varphi_2(0)}{A_1(1)}\right)-$$

$$-\frac{A_1(x)}{A_1(1)}\int_0^1\left(B(t)\tau(t)-f(t)\right)\frac{A_1(t)}{A_1'(t)}dt+\int_0^x\left(B(t)\tau(t)-f(t)\right)\frac{A_1(t)}{A_1'(t)}dt+\varphi_1(0) \qquad (24)$$

where

$$A_1(x)=\int_0^x exp\left(\int_0^t A(z)dz\right)dt \qquad (25)$$

Further, considering (21) and using (3) from (24) we will get:

$$\tau(x)=A_1(x)\left[\int_0^x A_1'(t)B(t)\tau(t)dt-\frac{\Gamma(\alpha)}{2}\int_0^x\frac{Q(t)A_1'(t)}{2a(t)+1}\frac{\lambda(t)}{\mu(t)}dt\int_0^t(t-s)^{\delta-1}\tau(s)ds\right]+$$

$$-\frac{A_1(x)}{A_1(1)}\int_0^1 \frac{A_1(t)}{A_1'(t)}B(t)\tau(t)dt+\int_0^x \frac{A_1(t)}{A_1'(t)}B(t)\tau(t)dt-$$
$$-\Gamma(\alpha)\frac{A_1(x)}{A_1(1)}\int_0^1 \frac{A_1(t)}{A_1'(t)}dt\int_0^t\left[\frac{Q(t)(t-s)^{\delta-1}}{2(2a(t)+1)}\frac{\lambda(t)}{\mu(t)}\right]\tau(s)ds-$$
$$-\Gamma(\alpha)\int_0^x \frac{A_1(t)}{A_1'(t)}dt\int_0^t\left[\frac{Q(t)}{2(2a(t)+1)}\frac{\lambda(t)}{\mu(t)}(t-s)^{\delta-1}\right]\tau(s)ds+f_1(x) \qquad (24)$$

where

$$f_1(x)=\left(1-\frac{A_1(x)}{A_1(1)}\right)\int_0^x \frac{2\Gamma(\alpha)d(t)A_1(t)}{A_1'(t)(2a(t)+1)}\frac{\lambda(t)}{\mu(t)}dt+2\Gamma(\alpha)A_1(x)\int_0^x \frac{d(t)A_1'(t)}{2a(t)+1}\frac{\lambda(t)}{\mu(t)}dt-$$
$$-\frac{A_1(x)}{A_1(1)}\int_0^x \frac{2\Gamma(\alpha)d(t)A_1(t)}{A_1'(t)(2a(t)+1)}\frac{\lambda(t)}{\mu(t)}dt-\frac{A_1(x)}{A_1(1)}\big(\varphi_2(0)-\varphi_1(0)\big)+\varphi_1(0) \qquad (25)$$

After some simplifications (24) we will rewrite on the form:

$$\tau(x)=A_1(x)\int_0^x \tau(t)\left[A_1'(t)B(t)-\frac{\Gamma(\alpha)}{2}\int_t^x \frac{Q(s)A_1'(s)}{2a(s)+1}\frac{\lambda(s)}{\mu(s)}(s-t)^{\delta-1}ds\right]dt+$$
$$+\int_0^x \frac{A_1(t)}{A_1'(t)}B(t)\tau(t)dt-$$
$$-\frac{\Gamma(\alpha)}{2}\int_0^x \tau(t)dt\int_t^x \frac{A_1(s)}{A_1'(s)}\frac{Q(s)}{2a(s)+1}\frac{\lambda(s)}{\mu(s)}(s-t)^{\delta-1}ds-$$
$$-\frac{A_1(x)}{A_1(1)}\int_0^1 \frac{A_1(t)}{A_1'(t)}B(t)\tau(t)dt\text{ -}$$
$$-\Gamma(\alpha)\frac{A_1(x)}{A_1(1)}\int_0^1 \tau(t)dt\int_t^1 \frac{A_1(s)}{A_1'(s)}\left[\frac{Q(s)(s-t)^{\delta-1}}{2(2a(s)+1)}\frac{\lambda(s)}{\mu(s)}\right]ds+f_1(x)$$

i.e. totally, we have integral equation:

$$\tau(x)=\int_0^1 K(x,t)\tau(t)dt+f_1(x). \qquad (26)$$

Here

$$K(x,t)=\begin{cases}K_1(x,t); & 0\le t\le x,\\ K_2(x,t); & x\le t\le 1.\end{cases} \qquad (27)$$

$$K_1(x,t)=\frac{\Gamma(\alpha)}{2}\int_x^t \frac{\lambda(s)Q(s)(t-s)^{\delta-1}}{\mu(s)(2a(s)+1)}\left(A_1(x)A_1'(s)+\frac{A_1(s)}{A_1'(s)}\right)ds+$$
$$+\left(A_1(x)A_1'(t)+\frac{A_1(t)}{A_1'(t)}-\frac{A_1(x)A_1(t)}{A_1(1)A_1'(t)}\right)B(t)-$$
$$-\Gamma(\alpha)\frac{A_1(x)}{A_1(1)}\int_t^1 \frac{A_1(s)}{A_1'(s)}\frac{Q(s)(s-t)^{\delta-1}}{2(2a(s)+1)}\frac{\lambda(s)}{\mu(s)}ds, \qquad (28)$$

$$K_2(x,t) = -\frac{A_1(x)A_1(t)}{A_1(1)A_1'(t)}B(t) - \Gamma(\alpha)\frac{A_1(x)}{A_1(1)}\int_t^1 \frac{A_1(s)}{A_1'(s)}\frac{Q(s)(s-t)^{\delta-1}}{2(2a(s)+1)}\frac{\lambda(s)}{\mu(s)}ds. \quad (29)$$

Owing to class (18), (19) of the given functions and after some evaluations from (28), (29) and (25), (27) we will conclude, that, $|K(x,t)| \le const$, $|f_1(x)| \le const$.

Since kernel $K(x,t)$ is continuous and function in right-side $F(x)$ is continuously differentiable, solution of integral equation (26) we can write via resolvent-kernel:

$$\tau(x) = f_1(x) - \int_0^1 \Re(x,t) f_1(t)dt, \quad (30)$$

where $\Re(x,t)$ is the resolvent-kernel of $K(x,t)$.

Unknown functions $\nu^-(x)$ and $\nu^+(x)$ we will found accordingly from (10) and (11):

$$\nu^-(x) = \frac{Q(x)}{2(2a(x)+1)}\int_0^x (t-x)^{\delta-1}dt\int_0^1 \Re(t,s) f_1(s)ds + \frac{Q(x)}{2(2a(x)+1)}\int_0^x (t-x)^{\delta-1} f_1(t)dt +$$

$$+\frac{1-2b(x)}{2a(x)+1}f_1'(x) - \frac{1-2b(x)}{2a(x)+1}\int_0^1 \frac{\partial \Re(x,t)}{\partial x} f_1(t)dt - \frac{2c(x)}{2a(x)+1}f_1(x) +$$

$$+\frac{2c(x)}{2a(x)+1}\int_0^1 \Re(x,t) f_1(t)dt - \frac{2d(x)}{2a(x)+1}$$

and $$\nu^+(x) = \lambda(x)\nu^-(x).$$

Solution of the Problem I in the domain Ω^+ we write as follows [21]

$$u(x,y) = \int_0^y G_\xi(x,y,0,\eta)\varphi_1(\eta)d\eta - \int_0^y G_\xi(x,y,1,\eta)\varphi_2(\eta)d\eta + \int_0^1 G_0(x-\xi,y)\tau^+(\xi)d\xi -$$

$$-\int_0^y\int_0^1 G(x,y,0,\eta)p_1(\xi,\eta)\tau^+(\xi)d\xi d\eta$$

Here $$G_0(x-\xi,y) = \frac{1}{\Gamma(1-\alpha)}\int_0^y \eta^{-\alpha}G(x,y,\xi,\eta)d\eta,$$

$$G(x,y,\xi,\eta) = \frac{(y-\eta)^{\alpha/2-1}}{2}\sum_{n=-\infty}^{\infty}\left[e_{1,\alpha/2}^{1,\alpha/2}\left(-\frac{|x-\xi+2n|}{(y-\eta)^{\alpha/2}}\right) - e_{1,\alpha/2}^{1,\alpha/2}\left(-\frac{|x+\xi+2n|}{(y-\eta)^{\alpha/2}}\right)\right]$$

Is the Green's function of the first boundary problem Eq. (1) in the domain Ω^+ [27],

$$e_{1,\delta}^{1,\delta}(z) = \sum_{n=0}^{\infty}\frac{z^n}{n!\Gamma(\delta-\delta n)}$$

is the Wright type function [27].

Solution of the Problem I in the domain Ω^{-} will be found by the formulate (9). Hence, the Theorem 2 is proved.

Conclusion. Under certain conditions for given functions, the uniqueness and existence of the formulated problem is proved. Investigated problem generalizes a number of local problems with continuous and discontinuous gluing conditions.

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